Electric Field Coupling to Short Dipole Receivers for Cavity Mode Enabled Wireless Power Transfer

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***Abstract*—This work provides a method of wireless power transfer that uses the resonant modes of a metallic cavity to deliver power to a small dipole nearly anywhere within the structure. We derive an expression for the coupling coefficient between the $\vec{E}$-fields of the cavity mode and the dipole, and then validate the analytic model via finite element simulations. Lastly, we use the results for the coupling coefficient to predict the wireless power transfer efficiency as the dipole is moved throughout the chamber.**

I. Introduction

Typical wireless power transfer (WPT) configurations use coupled coil resonators to transfer power via magnetic fields [1]. One limitation is that source and receiver need to be close together to achieve efficient WPT (<1 coil diameter apart). An alternative WPT system [2] uses resonant modes of an enclosed metallic cavity to uniformly illuminate large portions of the structure with electromagnetic energy, which can be received nearly anywhere within the cavity. Thus, the volumes of space where WPT is efficient can be extended beyond conventional coupled-coil WPT systems. In [2], however, the cavity-to-receiver coupling via the electric field was neglected. Here, analytic calculation and Finite Element Method (FEM) simulation are used to investigate WPT via coupling of the electric field to a small dipole receiver. First, we derive an expression for the coupling coefficient, then use it to compute an upper bound on the expected WPT efficiency. These results provide a tool for rapid exploration of what efficiencies can be expected for a dipole receiver in a given orientation, interacting with a particular cavity mode.

II. Derivation of the Coupling Coefficient

We start with coupled mode theory (CMT) definitions, and while generic for now, these definitions will later be used to determine the coupling between a cavity resonator and a subwavelength dipole. Each resonator is defined to have resonant frequency and amplitude, ω_1, a_1 and ω_2, a_2 (with $\omega_{1,2} = 2\pi f_{1,2}$), respectively, and that they have the time dependence $\exp(j\omega_{1,2}t)$. The two resonators are coupled via a coupling coefficient: $\kappa_{12} = \kappa_{21}^* \equiv \kappa_e$ (here, * indicates the complex conjugate). Lastly, using CMT, $a_{1,2}$ are defined such that their stored energy is: Energy = $|a_{1,2}|^2$. Using these definitions, it can be shown [3] that power fed from resonator one into resonator two (P_{21}) must be equal to the time rate of change of energy in resonator two [4]:

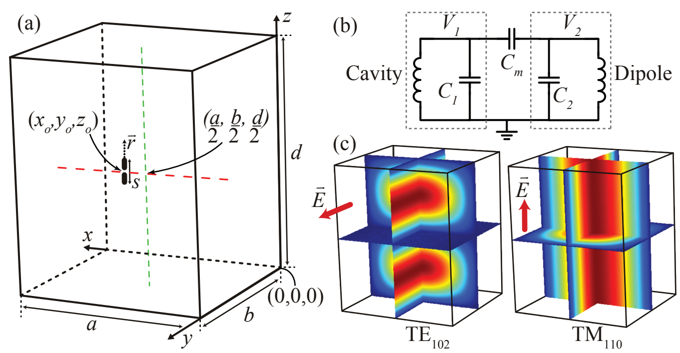

Fig. 1. (a) Setup of the cavity to dipole system analyzed in this work. (b) Simple circuit illustrating electric (capacitive) coupling between cavity resonator and resonant dipole. (c) Norm of electric field, $|\vec{E}|$, of modes analyzed in this work. Color: red, large; blue, small.

$$P_{21} = \frac{d}{dt}|a_2|^2 = j\kappa_e a_1 a_2^* - j\kappa_e^* a_1^* a_2 \tag{1}$$

The next step is to derive P_{21} for the cavity-to-antenna coupled mode system presented here. First, the power, P_{21}, flowing from resonator one (the cavity mode) into resonator two (the dipole with additional inductor such that an LC resonator is formed) is derived. The physical setup is shown in Fig. 1(a) for a cavity with dimensions $a \times b \times d$ and a dipole contained within that has length S and axis $\vec{r}$. Here coupling via the magnetic field will be neglected. First, note that the general capacitive coupling between cavity and dipole resonators can be captured via the simple circuit model in Fig. 1(b), where C_m is an abstract element present to capture capacitive coupling process via electric fields. Thus the power flowing from the chamber to the dipole can be written in terms of the charges of each of the capacitors:

$$P_{21} = v_2 C_m \frac{d(v_1 - v_2)}{dt} = \frac{\sigma_2}{C_2}\frac{d(\sigma_1 - \sigma_2)}{dt} \tag{2}$$

where $\sigma_{1,2}$, is the total charge on each of the resonators' effective capacitors, $C_{1,2}$. In the rightmost expression in (2) we have used $\sigma_2 = C_2 v_2$, where C_2 is the effective capacitance looking into the feed-point of the dipole (the dipole reactance looks capacitive if it is small compared to a wavelength, as in this work).

Next, $\sigma_{1,2}$ is reformulated in terms of $q_{1,2}$ – the time dependent complex envelope functions of the charges:

$$\sigma_{1,2}(t) = \frac{q_{1,2}e^{j\omega_{1,2}t} + q_{1,2}^* e^{-j\omega_{1,2}t}}{2} \tag{3}$$

Then, substituting (3), into (2), and then making the assumption that $\frac{d}{dt}q_{1,2} << j\omega q_{1,2}$, the result is:

$$P_{21} = \frac{1}{4C_2}(j\omega_1 q_1 e^{j\omega_1 t} q_2^* e^{-j\omega_2 t} - j\omega_1 q_1^* e^{-j\omega_1 t} q_2 e^{j\omega_2 t}) \quad (4)$$

Note that the form of (4) is similar to that of (1). Thus, κ_e can be obtained by inspection if we write our generic resonator amplitudes, $a_{1,2}$ from above, in terms of $q_{1,2}$. It is via algebraic manipulation that (4) can be made to match the form of (1), thus revealing κ_e. To make this substitution, we first require that the total energy stored in resonator one and two to be $|a_{1,2}|^2$, as was mentioned. We can accomplish this by introducing three parameters: α_e, the total electric energy stored in the chamber, β_e the total charge induced on the dipole due to the chamber's electric fields, $\vec{E}$, and ζ_e, a constant relating to the energy stored by the dipole when it is in isolation (i.e. charge stored on C_2). These parameters are:

$$\alpha_e = \iiint_V \epsilon_o |\vec{E}|^2 \, dV, \;\; \beta_e = C_2 \int_{l=\frac{-d_e}{2}}^{l=\frac{d_e}{2}} \vec{E}\cdot\vec{r}dl, \;\; \zeta_e = \frac{1}{\sqrt{2C_2}} \quad (5\text{–}7)$$

Here, V is the volume of the chamber, l is the line running from top to bottom of the dipole, and d_e is the effective dipole length, $d_e = S/2$ [5] (note this approximation means the dipole should be much smaller than a wavelength), and ϵ_o is the permittivity of air. Using these parameters, we can normalize $a_{1,2}$ such that $|a_{1,2}|^2$ gives the total energy stored in the cavity and resonant dipole-with-inductor, respectively. The explicit expressions for $a_{1,2}$ are:

$$a_1 = q_1 \frac{\alpha_e^{1/2}}{\beta_e} e^{j\omega_1 t}, \;\; a_2 = q_2 \zeta_e e^{j\omega_2 t} \quad (8)$$

After the above, (8) can be substituted into (4):

$$P_{21} = j\frac{\omega_1}{4C_2}\frac{\beta_e}{\alpha_e^{1/2}}\frac{1}{\zeta_e} a_1 a_2^* - j\frac{\omega_1}{4C_2}\frac{\beta_e}{\alpha_e^{1/2}}\frac{1}{\zeta_e} a_1^* a_2 \quad (9)$$

Now that (9) is in the same form as (1), the coupling coefficient between the cavity mode and a loop receiver (κ_e) can be determined by inspection:

$$\kappa_e = \frac{1}{4C_2}\frac{\omega_1 \beta_e}{\alpha_e^{1/2}\zeta_e} = \frac{\sqrt{2}}{4}\frac{\omega_1\sqrt{C_2}\beta_e}{C_2\sqrt{\alpha_e}} \quad (10)$$

Lastly, in [1] and [2], it was shown how maximum possible WPT efficiency, η_{max}, can be determined by knowing only κ_e and the Quality factors (Q-factors) of the two resonators (chamber and receiver), $Q_{1,2}$ (given a perfectly lossless bi-conjugate impedance matched system):

$$\eta_{max} = \frac{\chi\sqrt{1+\chi}}{(1+\sqrt{1+\chi})(1+\chi+\sqrt{1+\chi})}$$
$$\chi = \frac{4Q_1Q_2|\kappa_e|^2}{\omega_1\omega_2} \quad (11)$$

Thus, κ_e from above, along with knowledge of the Q-factors of the chamber and receiver allow for full prediction of the WPT efficiency.

III. Results

To verify our analytic model, we performed eigenvalue FEM simulations using COMSOL Multiphysics to determine the coupling coefficient numerically as a small dipole was moved throughout a chamber of dimensions $a = 1.52$ m, $b = 1.42$ m, and $d = 1.83$ m. Two modes were chosen to be investigated:

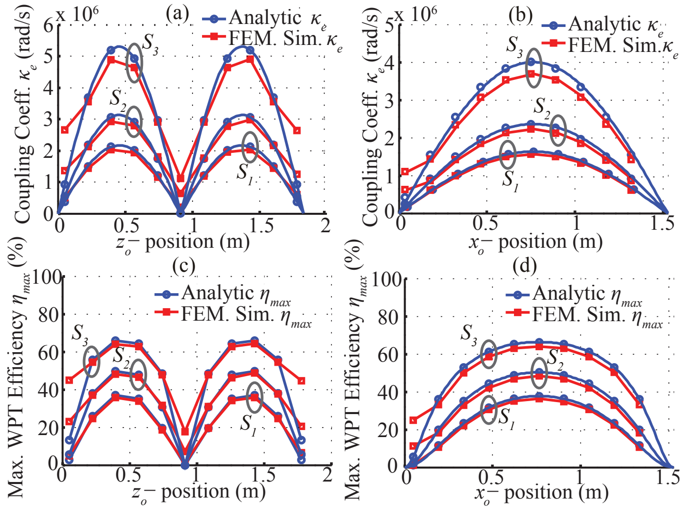

Fig. 2. Comparison of analytically computed and FEM simulated κ_e for (a) TE_{102} and, (b) TM_{110} modes and for varied dipole lengths, S_{1-3}. (c) and (d) are maximum possible WPT efficiency, η_{max} for TE_{102} and TM_{110}, respectively.

TE_{102}, and TM_{110}. Plots of the $\vec{E}$-fields for these modes are shown in Fig. 1(c). Three dipole lengths were chosen: S_1 = 3.81 cm, S_2 =5.08 cm, and S_3 =7.62 cm. For the TE_{102}, and TM_{110} modes, respectively, the dipole had $\vec{r} = \vec{a}_y$, $\vec{a}_z$. The dipole, with center position denoted as (x_o, y_o, z_o) was moved along the red dashed line of Fig. 1(a) for the TE_{102} mode, and along the green dotted line for the TM_{110} mode.

The results comparing the FEM simulations to the analytic calculation are shown in Fig. 2(a) and (b). They show good agreement between analytically and numerically computed κ_e, with typically <8% error. Finally, Fig. 2(c) and (d) show the maximum possible WPT efficiency that can be obtained by using κ_e from Fig. 2(a) and (b) along with the realistic values [2] of cavity mode and receiver Q-factors of Q_1=1000, Q_2=300 in (11). Note that the maximum possible values exceed 60% for the longer dipoles studied; however, the dipoles are still quite small and so offer a method for compact receiver design in cavity mode enabled WPT without sacrificing efficiency.

IV. Conclusion

We derived an expression for the coupling coefficient between the $\vec{E}$-field of a resonant mode of a cavity resonator and a subwavelength resonant dipole receiver and showed good agreement between the analytic expression and FEM simulation. We then demonstrated how to predict the efficiency of WPT for a dipole interior to the chamber.

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